

Amusement Park Physics

Introduction

Notes to Teachers:

1. Copy and hand out a packet to each student.
2. Remind the students to help keep the park clean.
3. We suggest having the students answer all of the questions for six attractions.

Notes to Students:

1. A safe day is a fun day.
2. Items to bring:
 - ✓ Pencil
 - ✓ Paper
 - ✓ Activity packet
 - ✓ Accelerometers
 - ✓ Stop watch or watch with a second hand
 - ✓ Calculator

SAFETY NOTICE

Any instrument or devices carried on the rides by students should be made of plastic and have a wrist tether, so that if dropped the instrument will not break or fall off the ride and cause injury or damage.

Note: Information in this document was provided by Lane Smith from Grand Haven High School.

USEFUL FORMULAS

$$F = ma$$

$$E_p = mgh$$

$$E_k = \frac{1}{2}mv^2$$

$$mgh = \frac{1}{2}mv^2$$

$$v^2 = 2gh$$

$$v = \sqrt{2gh}$$

$$g = 9.8 \text{ m/s}^2$$
$$32 \text{ ft/s}^2$$

$$p = mv$$

$$\lambda = \frac{h}{p}$$

$$W = Fd$$

$$P = \frac{w}{t}$$

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$d = \left[\frac{v_i + v_f}{2} \right] t$$

$$d = v_i t + \frac{1}{2}at^2$$

$$v_f = v_i + at$$

$$v_f^2 = v_i^2 + 2ad$$

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$a = \frac{v^2}{r}$$

$$F = \frac{mv^2}{r}$$

$$a = \frac{4\pi^2 r}{t^2}$$

$$F = \frac{m4\pi^2 r}{t^2}$$

MAKING MEASUREMENTS

TIME

The times that are required to work out the problems can easily be measured using a watch with a second hand or a digital watch with a stopwatch mode. When measuring the period of a ride that involves harmonic or circular motion, measure the time for several repetitions of the motion, then divide by the number of repetitions. This will give a better estimate of the period of motion than just measuring one repetition. You may want to measure the time two or three times and then average them.

DISTANCE

Since you cannot interfere with the normal operation of the rides, you will not be able to directly measure heights, diameters, etc. All but a few of the distances can be measured remotely using the following methods. They will give you a reasonable estimate. Try to keep consistent units, i.e. meters, centimeters, etc., to make calculations easier.

Pacing

Determine the length of your stride by walking at your normal rate over a measured distance. Divide the distance by the number of steps and you can get an average distance per step. Knowing this, you can pace off horizontal distances.

My pace = _____ m

Ride Structure

Distance estimates can be made by noting regularities in the structure of the ride. For example, tracks may have regularly spaced cross-members as shown in figure a. The distance d can be estimated, and by counting the number of cross members, distances along the track can be determined. This method can be used for both vertical and horizontal distances.

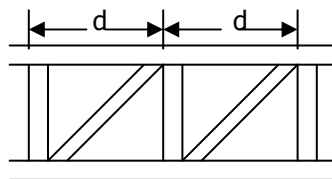


figure a

Triangulation

For measuring height by triangulation, an astrolabe such as that shown in figure b can be constructed. You can practice this with the school flagpole before coming to the park.

Suppose the height h_T of the Corkscrew must be determined.

1. Measure the distance between you and the ride. You can pace off the distance.

Distance d : _____ m

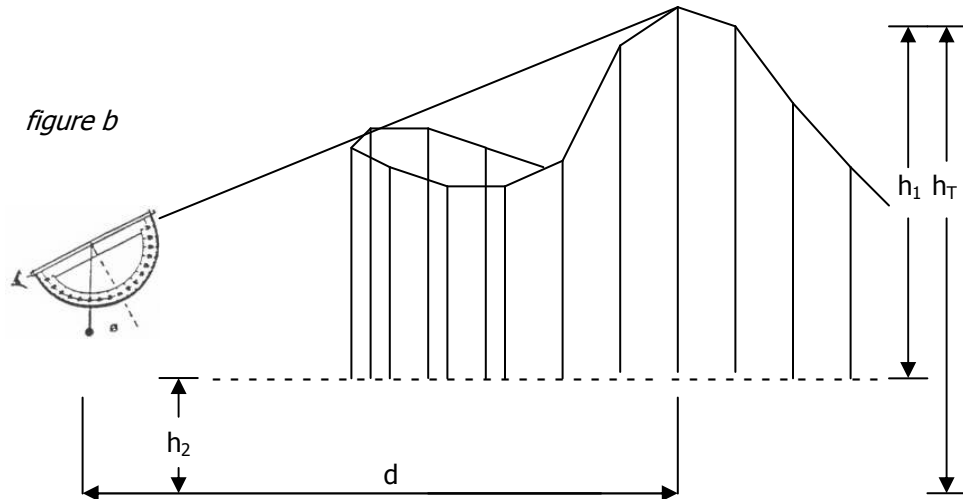
2. Measure the height of the string hole.

String hole height $h_2 = \underline{\hspace{2cm}}$ m

3. Take a sighting at the highest point of the ride.

4. Read off the angle of elevation.

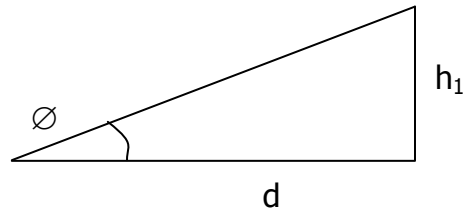
Angle of elevation $\underline{\hspace{2cm}}$



Then since

$$h_1/d = \tan \phi$$

$$h_1 = d(\tan \phi)$$



5. Look up the tangent value for the angle measured:

Tangent value $\underline{\hspace{2cm}}$

ANGLE	TANGENT	ANGLE	TANGENT	ANGLE	TANGENT
5	.09	35	.70	65	2.14
10	.18	40	.84	70	2.75
15	.27	45	1.00	75	3.73
20	.36	50	1.19	80	5.67
25	.47	55	1.43	85	11.43
30	.58	60	1.73	90	57.29

6. Multiply tangent value by the distance from the ride: $h_1 = \underline{\hspace{2cm}}$ m

7. Add this to the height of the string hole: $h_2 = \underline{\hspace{2cm}}$ m

8. This number is the height of the ride : $h_T = \underline{\hspace{2cm}}$ m

Other

There are other ways to measure distance. If you can think of one, use it. For example, a similar but more complex triangulation could be used. If you can't measure the distance L because you can't get close enough to the base of the structure, use the Law of Sines as in figure c below.

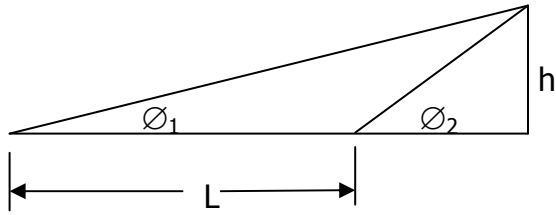


Figure c

Knowing ϕ_1, ϕ_2 , and L , the height h can be calculated using the expression

$$h = \left[\frac{\sin \phi_1 \sin \phi_2}{\sin (\phi_2 - \phi_1)} \right] L$$

ACCELERATION

Accelerometers are designed to record the **g** forces felt by a passenger. Accelerometers are usually oriented to provide force data perpendicular to the track, longitudinally along the track, or laterally to the right or left of the track (see Figure d).

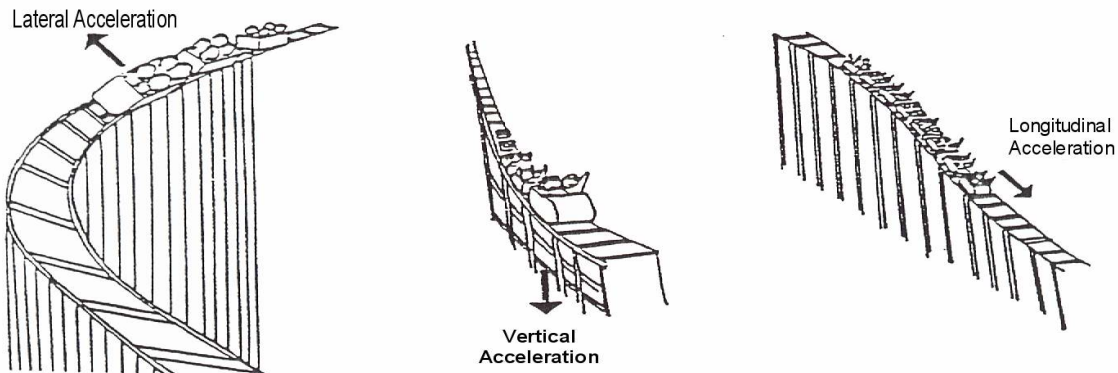


Figure d

Accelerometers are calibrated in g's. A reading of 1 g equals an acceleration of 9.8 m/s^2 . As you live on Earth, you normally experience 1 g of acceleration vertically (no g's laterally or longitudinally). Listed below are the sensations of various g forces. These are rough estimates, but may be helpful in estimating accelerations on the various rides.

Accelerometer Reading	Sensation
+3.0 g	3 times heavier than normal (maximum g's pulled by space shuttle).
+2.0 g	Twice normal weight.
+1.0 g	Normal Weight.
+0.5 g	1/2 Normal weight.
+0.0 g	Weightlessness (no force between rider and coaster).
- 0.5 g	1/2 Normal weight, but directed away from the coaster seat.

SPEED

In linear motion, the average speed of an object is given by:

$$v_{ave} = \frac{\Delta d}{\Delta t}$$

In circular motion, where speed is constant:

$$v_{ave} = \frac{\Delta d}{\Delta t} = \frac{\Delta \pi r}{\Delta t}$$

Both of these cases involve fairly constant speed. Be careful measuring speed when the speed is changing. If you want to determine the speed at a particular point on the track, measure the time it takes for the length of the train to pass that particular point. The train's speed is then given by:

$$v_{ave} = \frac{\Delta d}{\Delta t} = \frac{\text{length of train}}{\text{time to pass point}}$$

In a situation where it can be assured that total mechanical energy is conserved, the speed of an object can be calculated using energy considerations. Suppose the speed at point C is to be determined (see Figure e). From the principle of conservation of total mechanical energy it follows that:

$$PE_A + KE_A = PE_C + KE_C$$

$$mgh_A + \frac{1}{2}mv_A^2 = mgh_C + \frac{1}{2}mv_C^2$$

Since mass is constant, solving for v_C

$$v_C = \sqrt{2g(h_A - h_C) + v_A^2}$$

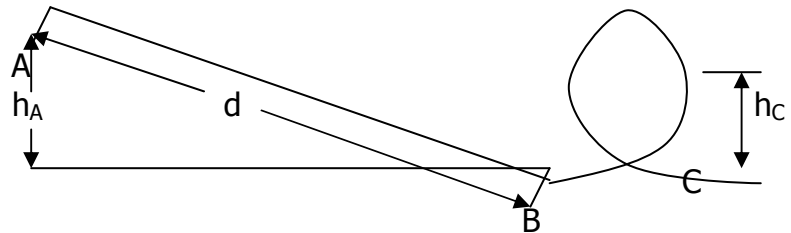
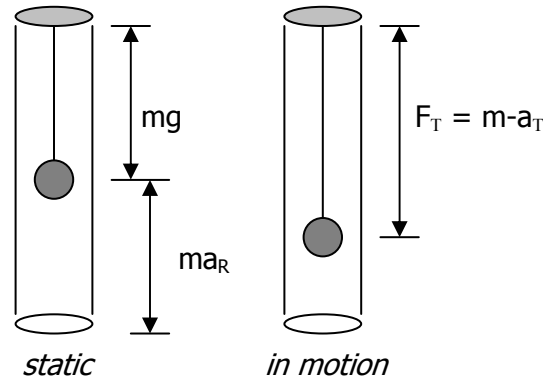


Figure e

Thus by measuring the speed of the train at point A, and the heights h_A and h_C , the speed of the train at point C can be calculated.

VERTICAL ACCELERATION

A simple device for measuring vertical accelerations is a 0-5 Newton spring scale with a 100g mass attached. The plastic tube with elastic and fishing weight approximate this equipment. The forces on the mass are drawn where F_T is the reading on the scale.



The forces on the masses are shown in the diagram.

If the person is holding the scale right side up, then:

$$F_T = mg + ma_{(\text{Ride})} \quad \text{or} \quad ma_{(\text{Total})} = mg + ma_{(\text{Ride})}$$

Since m is constant

$$a_T = g + a_R \quad \text{or} \quad a_R = a_T - g$$

If the person is holding the scale upside down against gravity as might be found at the top of a loop, then

$$a_R = -(a_T + g) \quad \text{ie. Acceleration is upwards}$$

In either situation, the acceleration can be calculated by knowing F_T (or a_T).

LONGITUDINAL ACCELERATION

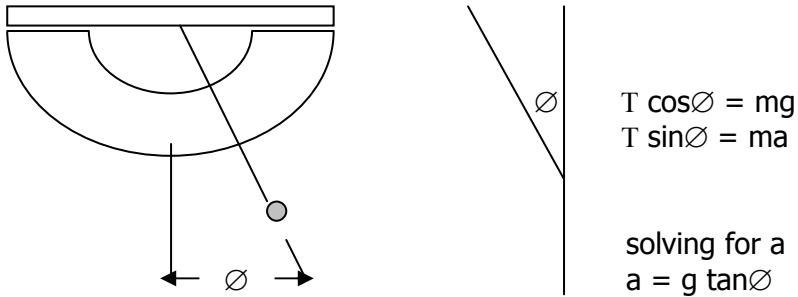
Acceleration of a person on a ride can also be determined by direct calculation. Down an incline, the average acceleration of an object is defined as:

$$a_{\text{ave}} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\text{change in speed}}{\text{change in time}}$$

Using methods previously discussed it is possible to estimate speeds at both the top and bottom of the hill and the time it takes for the coaster to make the trip. Thus average acceleration can be found during that portion of the ride.

LATERAL ACCELERATION

The astrolabe discussed earlier as a triangulation instrument may also be used to measure lateral accelerations. The device is held with the sighting tube horizontal, and weight swings to one side as you round a curve. By measuring the angle, acceleration can be determined. See the drawing below:



CENTRIPETAL ACCELERATION

With uniform circular motion remember that:

$$v = \frac{2\pi r}{t}$$

and the centripetal acceleration is given by:

$$a_c = \frac{v^2}{r} = \frac{4\pi^2 r}{t^2}$$

Thus centripetal acceleration can be measured on a ride.